United States Physics Team

Entia non multiplicanda sunt praeter necessitatem

1998 Creative Response Portion of Exam 1

4 Questions, 60 minutes

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

Show all work, as partial credit may be earned.

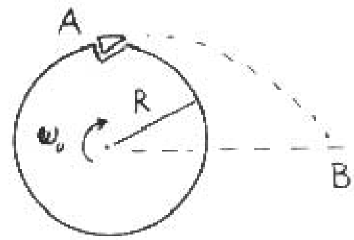
Communicate! Only that which is written can be graded.

A hand-held calculator may be used. Its memory must be cleared of data and programs. Calculators may not be shared.

Possibly useful information:

Gravitational field at Earth's surface: $g = 9.8$ N/kg
Newton's gravitational constant: $G = 6.67\times10^{-11}$ Nm^2/kg^2
Coulomb's constant: $k = 1/4\pi\epsilon_o = 9\times10^9$ Nm^2/C^2
Elementary charge: $e = 1.6\times10^{-19}$ C
Binomial expansion: $(1 + z)^n \approx 1 + nz$ whenever $|z| << 1$

1. **(25 points)** A disc of radius R spins with angular velocity ω_o about its axis, which is held horizontally in frictionless bearings. The disc's moment of inertia about the spin axis is I_o. At a certain instant, a small chip of mass m breaks off the disc and flies away moving tangent to the wheel. The chip breaks off when it is at the top of the disc (i.e., at "12:00 o'clock"), which is point A in the accompanying diagram. Please specify all answers in terms of the given quantities and known constants.



(a, 4 points) What is the initial velocity v_o of the chip at point A when it separates from the disc?

(b, 7) How far is the chip from the axis of the disc when it arrives at the level of the axis (point B on the diagram)?

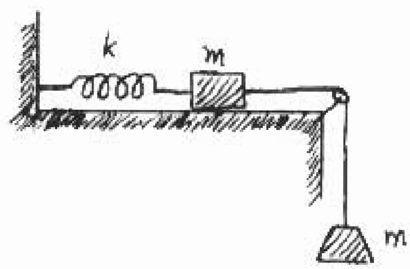
(c, 7) With what speed does the chip arrive at point B?

(d, 7) After the chip breaks off, what is the angular velocity of the disc?

2. (25 points)

(a, 5) A certain linear spring is stretched by the amount x_o = 20cm when a block of mass m = 1kg hangs vertically from it at rest. Find the spring constant k. (Throughout this entire problem, neglect the mass of the spring.)

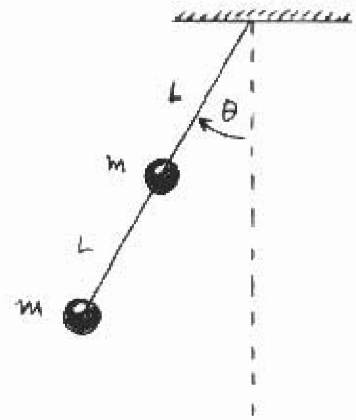
(b, 10) One end of this same spring is now fastened to a rigid vertical wall, and the block of mass m = 1 kg is attached to the other end of the spring such that this block rests on the surface of a horizontal table (see diagram). By means of strings and a pulley of negligible mass, another 1 kg mass hangs as shown. The system is released from a state of rest, with the spring initially relaxed. The coefficient of sliding friction μ between the block and the table has the value of ¼. Find the maximum stretch of the spring. Neglect air resistance.



(c, 10) If the friction between the block and the table were removed, what would be the period of the system's oscillation?

3. (25 points) Two small but heavy metal beads, each of mass m, are soldered to a thin rigid rod of length $2L$. One bead is soldered halfway along the rod, and the second bead is soldered at one end of the rod. The other end of the rod is connected to the ceiling by a frictionless pivot, making a pendulum of the rod and two beads (see the sketch). The mass of the rod is negligible, as is air resistance. This pendulum is pulled to one side such that it is initially horizontal ($\theta = 90°$). The pendulum is then released and swings down, with θ decreasing from $90°$ through $0°$.

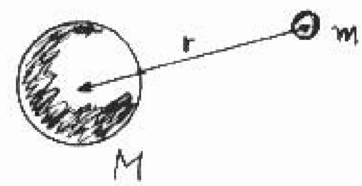
(a, 9) Find the speed of each bead when the rod is vertical ($\theta = 0°$), assuming for now that neither section of the rod breaks.



(b, 16) Either section of the rod will break if subjected to a tension force of $4.2mg$ or greater. In the motion described above, does either section break? If so, find the angle at which it breaks.

4. (25 points)

(a, 7) Use Newton's law of gravitation to determine the constant C in Kepler's third law, $T^2 = Cr^3$, where T is the period of a satellite of mass m in orbit about a planet of mass M, and r is the radius of the satellite's orbit (assumed circular for simplicity). Assume $M >> m$.



(b, 6) Find the orbital radius of a geosynchronous satellite (i.e., a satellite that orbits such that it is always above the same point relative to the surface of the Earth).

Possibly useful data:

Mass of Earth: 5.98×10^{24} kg

Radius of Earth: 6.37×10^{6} m

(c, 6) Consider a communications satellite in geosynchronous orbit above the mouth of the Amazon River (which is on the equator). If the radius of the satellite's orbit decreases by 2 km, what is the change in the period (including the correct sign) of the satellite's orbit?

(d, 6) For the satellite of part (c), after its orbit radius decreases by 2 km, what is the velocity, relative to an observer standing at the mouth of the Amazon, of the spot on the surface of the Earth that is directly beneath the satellite?